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Use of Analytical Modeling
and Limited Data for Prediction
of Mesoscale Eddy Properties

by

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ABSTRACT

The use of analytical modeling in the study of oceanic eddies is considered. Limited observational data, in combination with eddy models, can be used to obtain analytical approximations to environmental effects, including current and temperature perturbations, throughout the eddy. Techniques which efficiently use discrete measurements are presented to accurately specify any given analytical model containing an arbitrary number of parameters to an observed eddy. Questions of unique parameter specification and data sufficiency are considered for various data types and amounts, using a previously-derived eddy model. Examples with bathythermograph data are presented, in which eddy size, strength, and center position are to be determined. AXBT data is emphasized, and an investigation is made of the influence of the number of such instruments on the accuracy of parameter estimates. It is then shown how data obtained from oceanographic moorings can also lead to specification of eddy drift speed and direction. In both the bathythermograph and mooring examples, it is demonstrated that, even when the type of data available leads to nonunique parameter specification, significant information can be obtained about the observed eddy. Results in this paper suggest certain efficiencies in data utilization and in the design of subsequent experiments.

1. Introduction

Oceanic eddies have been observed to be responsible for significant environmental perturbations in the ocean's static state. Large elevations or depressions in the normally horizontal isotherms have been observed, resulting from eddy rotational currents, and the translation of these masses of cold or warm water has been postulated as playing a significant role in the energy balance of the oceans (Lai and Richardson, 1977). Investigations of individual eddies have typically employed extensive deep bathythermograph readings, to depths of several thousand meters at frequent horizontal locations (see, for example, Fuglister, 1971). Other studies have also incorporated current data obtained from moorings (see for example, Koshlyakov and Grachev, 1974) in the examination of eddies in mid-ocean regions. In such experiments, the resulting discrete readings are used to approximate the continuum of eddy environmental effects by either simple smoothing of the data, or the more sophisticated techniques of objective analysis proposed by Bretherton, Davis, and Fandry (1976), and implemented by Freeland and Gould (1976). However, these procedures typically require a high density of readings throughout the oceanic region containing the eddy, requiring fairly extensive ship time and equipment. Further, results from these procedures are restricted to regions within the eddy where data was acquired, so that they do not in general determine eddy effects at all horizontal positions and depths.

In attempts to explain and describe eddy structure, analytical research has produced several eddy models. Stern (1975) considered variational principles for equilibrium of barotropic eddies, with possible extensions to the baroclinic case. Andrews and Scully-Power (1976) proposed a model for the velocity fields and dynamic heights of an anticyclonic eddy. Flierl discussed both the use of nonlinear dynamics to study eddies linked to the barotropic shear flow (Flierl, 1976), and linear dynamics to study the evolution and translation of eddy-like disturbances (Flierl, 1977). The above studies did not attempt to compare theoretical structures, such as those of temperature and current, to those of observed eddies. However, a recent model of several of the authors (Henrick, Siegmann, and Jacobson, 1977) was shown to give satisfactory agreement to current and temperature perturbations observed in one Gulf Stream ring. The model is applicable to a class of oceanic eddies satisfying certain assmmptions. We remark that any eddy model will necessarily reflect the ultimate objectives of the modeler. For example, studies of biological activity in Gulf Stream rings (see, for example, Wiebe et al, 1976) might require high accuracy in predicting near-surface effects, while long-range acoustical propogation studies might require knowledge of eddy influences at great depths. We note also that no one model could be expected to appropriately describe all eddies, nor could it completely describe one particular eddy from its nascent to terminal stages.

We consider here the problem of utilizing analytical models and limited data in the description of properties of an observed eddy. If the environmental effects of an eddy could be approximated to the accuracy required for a particular application, then estimation of model parameters, using the model and limited data, would be an efficient procedure. Indeed, such an approach might require significantly less extensive measurements than presently used, while simultaneously avoiding the known hazards of ocean undersampling. Of course, we caution against the use of an eddy model without adequate justification or verification, since this could prejudice the ocean measurements and lead to grossly inaccurate descriptions. In this paper, we also make the important assumption that enough evidence is available to warrant application of the particular model chosen. Our purpose here is to specify the types and amounts of data necessary for model-parameter specification, and to present some techniques to implement such a procedure.

In Sec. 2, a brief review of a previously derived eddy model is presented. The assumptions implicit in this model are outlined and the model parameters, including eddy size, depth, direction and speed of rotation, and drift trajectory, are presented. The problem of using observational data to specify the parameters implicit in any eddy model is discussed in Sec. 3. The question of the minimum amount of required data, a necessary prerequisite for accurate parameter specification, is discussed. Then, a technique for accurately specifying model parameters, using more than the minimal data, is presented. These procedures are next applied to the particular model of Sec. 2. Sec. 4 discusses the use of temperature and current data in selecting model parameters, and the processing of such data for use in the parameter-specification process. Several distinct varieties of such data are then used to specify model parameters, for the model of Sec. 2 as an example. The parameter-fitting technique is first applied in Sec. 5 to time-independent data obtained from bathythermographs. It is shown that if data through the eddy is taken in a path lying in a vertical plane, ambiguity in the position of the eddy center can result, while inclusion of nonplanar data leads to unique parameter values. Sec. 6 then investigates the use of oceanographic moorings to study both eddy structure and translation. Nonuniqueness of parameter values results if only a single thermistor string is used, but inclusion of additional thermistor strings, or current meters, results in unique parameter specification. The principal results of the paper are summarized in Sec. 7.

2. An eddy model

Using the basic equations of fluid motion, Watson, Siegmann, and Jacobson (1976) developed a method for analytically determining approximate environmental effects of a class of mesoscale flows, which include large oceanic eddies. From these results, an analytical model for eddies nearly in geostrophic and hydrostatic balance was derived (Henrick, Siegmann, and Jacobson, 1977), which we now summarize briefly in revised form.

Approximations to the static state of an ocean, with horizontal surface and bottom separated by distance D, can be obtained from the fluid equations by setting the velocity field equal to zero. The result represents a vertically-stratified fluid in hydrostatic balance, with static density ρ_s and pressure P_s given by

$$\rho_s(z) = \rho_o + \rho_o\, g\, z/c_o^2 + \pi(z) \tag{1a}$$

and

$$P_s(z) = g \int_0^z \rho_s(r)dr\ . \tag{1b}$$

In (1), z is depth measured positively from the ocean surface, ρ_o is the surface density, g is gravitational acceleration, c_o the surface sound speed, and $\pi(z)$ is potential density. We select π to have the observationally reasonable and analytically convenient form

$$\pi(z) = \rho_o\, A[1-(1+Bz)^{-1}]\ , \tag{2}$$

where A and B are constants. The values of ρ_o, c_o, A, and B are chosen by fitting oceanographic data from the area under consideration. For example, if data on temperature T is available, a useful state equation is one proposed by Eckart (1958), relating density ρ and pressure P in the form

$$F(\rho,P,S,T) = 0\ . \tag{3}$$

If salinity S is assumed constant at 35 ^{o}/oo, (3) can be solved (Baer and Jacobson, 1974) for temperature as a function of ρ and P. Then, values of ρ_o, A, and B can be determined by fitting the theoretical temperature structure to the observed data. For instance, by utilizing data from near Eleuthera, we obtained the typical values of $\rho_o = 1.02322$ gm/cm^3, A = 0.00634, and B = 2.312 km^{-1}.

To determine motion-induced perturbations to the static state, a right-handed Cartesian coordinate system is defined at some latitude ϕ, with the x-axis oriented positively to the east and y-axis to the north. Appropriate scaling of the fluid equations leads to simplifications valid for parameter ranges that may include mesoscale eddies. Motion-induced perturbations $\bar{\rho}$ and $\bar{P}$ to the static pressure and density fields are sought, so that the total pressure and density may be written as

$$P = P_s(z) + \bar{P}(x,y,z,t) \tag{4a}$$

and

$$\rho = \rho_s(z) + \bar{\rho}(x,y,z,t), \tag{4b}$$

where t represents time. The resulting horizontal current $\underset{\sim}{V} = (u,v)$ can be written in terms of a stream function ψ as

$$u = -\psi_y \tag{5a}$$

and

$$v = \psi_x , \tag{5b}$$

while density and pressure perturbations are given in terms of ψ by

$$\bar{\rho} = (2\Omega\rho_o g^{-1} \sin \phi)\ \psi_z \tag{5c}$$

and

$$\bar{P} = (2\rho_o\Omega\ \sin\phi)\ \psi\ , \tag{5d}$$

where Ω represents the earth's angular velocity. The stream function is then constrained to follow a time-independent quasi-geostrophic potential vorticity equation, which is equivalent to the conservation of potential vorticity following a material particle. The quasi-geostrophic balance hypothesis has been used by other eddy modelers (see, for example, Stern, 1975), and was tested in the *MODE experiment* (Bryden, et al, 1975), although the results were inconclusive.

Solutions that exhibit eddy-like characteristics are then sought to the potential vorticity equation. Since eddies are often observed to be nearly circular features drifting through the ocean (see, for example, Fuglister, 1971), approximate solutions are considered which are radially symmetric about a central axis and which translate with time along some path with assumed functional form $[x_o(t),\ y_o(t)]$. Hence, the variable

$$r^2 = [x - x_o(t)]^2 + [y - y_o(t)]^2 \tag{6a}$$

is introduced, and the stream function is written as

$$\psi(x,y,z,t) = \chi(r,z). \tag{6b}$$

A reasonable set of boundary conditions can be postulated from observed eddy features. For example, eddy currents and perturbation pressure and density are required to vanish at some finite distance r_o from the eddy center and at some depth z_o ($\leq D$), below which eddy effects are assumed negligible. Other assumptions, physically motivated from surface conditions of unidirectional rotational currents, finite pressure, and vanishing vertical velocity, lead to additional requirements for the solution. Finally, the maximum surface current speed is normalized to the velocity scale U_o. An approximate solution to the boundary value problem is then obtained by superimposing the vertical barotropic and first baroclinic modal solutions. The resulting solution (Henrick, Siegmann, and Jacobson, 1977) is

$$\chi(r,z) = K_1 U_o r_o [J_o(\alpha_1 r/r_o) - J_o(\alpha_1)][F(z) - F(z_o)], \qquad (7a).$$

where

$$K_1 = \pm Dg\{2\alpha_1 \alpha_2 c_o \Omega \sin\phi \; [F(0) - F(z_o)]\}^{-1} , \qquad (7b)$$

$$F(z) = (1+Bz)^{-1/2}\{\cos[(\gamma/2)\ln(1+Bz)] + (1/\gamma)\sin[(\gamma/2)\ln(1+Bz)]\} , \qquad (7c)$$

and

$$\gamma = 2\pi/\ln(1+Bz_o) . \qquad (7d)$$

In (7), J_o is the Bessel function of the first kind and order zero, $\alpha_1 \doteq 3.83$ is the first zero of the Bessel function J_1 of the first kind and order one, $\alpha_2 \doteq .582$ is the maximum of J_1, and ln is the natural logarithm. In (7b) the plus(minus) sign corresponds to an anticyclonic (cyclonic) eddy with clockwise (counterclockwise) rotational velocity. We note that both the separability of horizontal and vertical dependence and the superposition of baroclinic and barotropic modes, employed in the development of this model, are characteristic of other eddy models (see, for example, Flierl, 1976).

Resulting velocity, density, and pressure fields can be determined through (5)-(7). With density and pressure thus determined at any point in the eddy, temperature can then be obtained from the inverted Eckart equation (3) when $S = 35\ ^o/oo$. In order to determine eddy-environmental effects, the parameters characterizing the eddy must be specified. In the particular model described above, we need to choose the radius r_o, depth of influence z_o, maximum current speed U_o, direction of rotation [the sign in (7b)], and the translational trajectory described by $x_o(t)$ and $y_o(t)$.

We note that the above model has been shown to agree fairly well with the observed features of one Gulf Stream ring, and that it will be used as an example in subsequent sections. However, it does not admit such possibly significant features as radial asymmetry, salinity variation, or near-surface effects. Nonetheless, these limitations do not affect the general objectives of this paper, since any eddy model which contains an arbitrary number of parameters may be utilized in the procedures to be discussed.

3. Parameter determination

Using observational data to specify values of parameters in an appropriate eddy model requires the matching of observed perturbations in the static ocean state with eddy effects predicted by the model. We consider here the problem of using eddy-induced environmental deviations of general types to accurately select model parameters.

a. Minimum-data parameter specification

The matter of determining how little data of a certain type is necessary to specify a given eddy model has obvious theoretical and experimental importance. By examining the minimum-data problem, much can be learned about the analytical dependence of the model on its parameters. Moreover, knowledge of the minimum data set is a necessary prerequisite to determining the amount of data sufficient to accurately specify model parameters.

Any model that specifies eddy-environmental effects depends on position and time, and is assumed to include n parameters $\underset{\sim}{p} = (p_1,\ldots,p_n)$. If values for these parameters can be selected, eddy-environmental effects at any point are then known to the accuracy of the model. For fixed location and time, the eddy model can be regarded as specifying eddy effects as nonlinear functions of the n parameters. Suppose a deviation Δ from the ocean's static state is observed at a particular position and time (x_1,y_1,z_1,t_1). A corresponding relation of the form

$$M = f(x_1, y_1, z_1, t_1; \underset{\sim}{p}) \tag{8}$$

for the deviation M predicted by the model equations can be obtained, where the components p_j of $\underset{\sim}{p}$ are viewed as independent variables. It is tempting to suppose that given a sufficient number ($\geq n$) of observations, a unique choice of the n parameters could be determined by solving equations (8) with each M replaced by the corresponding Δ at n distinct positions or times. However, due to the functional forms of (8), it is possible that these measurements may not be taken at arbitrary points in space or time. Moreover, unique parameter values may not result, even if a proper sampling is performed. Thus, the problem of parameter determination requires further consideration.

We suppose here that the model equations at any (x,y,z,t) are separable into a product of vertical and horizontal functions, each with distinct parameters:

$$M = f_H(x,y,t; \underset{\sim}{p}_H)\ f_V(z\ t; \underset{\sim}{p}_V)\ . \tag{9}$$

In (9), $\underset{\sim}{p}_H$ represents the vector of n_H parameters $p_H^{(i)}$, $1 \leq i \leq n_H$, associated with the description of the horizontal structure and translation of the eddy. Similarly, $\underset{\sim}{p}_V$ is the vector of n_V parameters $p_V^{(i)}$, $1 \leq i \leq n_V$, which describe the eddy vertical structure. Such separability is characteristic of many eddy models, and appears justifiable from observations of near-uniform decay of eddies with depth. However, if such separability were not permitted in the model, it would be necessary to choose all the parameters at once.

Suppose that deviations in the ocean's static state $\Delta_1^{(j)}$ are observed at a particular fixed horizontal position and time (x_1, y_1, t_1) and at $n_V + 1$ different depths $z_1^{(j)}$, $1 \leq j \leq n_V + 1$. The vertical parameters can be determined by equating observed and predicted environmental deviations for each j, giving

$$\Delta_1^{(j)} = f_H(x_1, y_1, t_1; \underset{\sim}{p}_H)\ f_V(z_1^{(j)}, t_1; \underset{\sim}{p}_V),\ 1 \leq j \leq n_V + 1\ . \quad (10)$$

The horizontal function f_H, which is constant at all depths, can then be eliminated from these equations by solving for f_H at a single depth, and substituting into the remaining n_V equations. This results in n_V equations of the form

$$\Delta_1^{(j)} f_V(z_1^{(1)}, t_1; \underset{\sim}{p}_V) - \Delta_1^{(1)} f_V(z_1^{(j)}, t_1; \underset{\sim}{p}_V) = 0,\ 2 \leq j \leq n_V + 1\ , \quad (11)$$

where, without loss of generality, we have chosen the first equation to eliminate horizontal dependence. Approximate solutions to these equations for the n_V vertical parameters can then be obtained numerically, and the vertical structure of the eddy completely specified.

Knowledge of the vertical structure can be used to eliminate vertical dependence of measurements at any point in the eddy. If a measurement $\Delta_1^{(j)}$ is taken at any horizontal position (x_1, y_1) and time t_1, and depth $z_1^{(j)}$, depth dependence can be eliminated by defining the vertically-scaled environmental deviation

$$\bar{\Delta}_1^{(j)} = \Delta_1^{(j)}/f_V(z_1^{(j)}, t_1; \underline{p}_V), \ 1 \le j \le m \ , \tag{12}$$

where typically measurements would be available at m depths $z_1^{(j)}$, $1 \le j \le m$. The magnitude of eddy effects at (x_1, y_1, t_1) can then be determined by

$$\bar{\Delta}_1 = (1/m) \sum_{j=1}^{m} \bar{\Delta}_1^{(j)} \ , \tag{13}$$

where $\bar{\Delta}_1$ is the average vertically-scaled environmental deviation. This averaging procedure avoids propagation of errors in data at any one depth, and hence allows more accurate measurement of the strength of eddy effects at (x_1, y_1, t_1). We note that if the vertical model structure fits the observed vertical structure exactly, then the terms in sum (13) are identical.

The horizontal parameters can then be chosen using the vertically independent measurements from (13). Given observations at n_H distinct horizontal positions and/or times (x_i, y_i, t_i), $1 \le i \le n_H$, where at least one of the x_i, y_i, or t_i are distinct for each i, the average vertically-scaled environmental deviation $\bar{\Delta}_i$ can be determined at each position as in (13). Each $\bar{\Delta}_i$ can then be equated with the vertically-scaled deviation predicted by the model, resulting in n_H equations in the form

$$\bar{\Delta}_i = f_H(x_i, y_i, t_i; \underset{\sim}{p}_H) \tag{14}$$

for the n_H unknown parameters. Approximate solutions for the horizontal parameters can then be obtained numerically, resulting in specification of horizontal eddy structure. However, here and in the vertical problem (11) it is possible that exact solutions might not exist, since any given model may not exactly match observed effects, which themselves contain experimental inaccuracies. This problem can be alleviated by using procedures outlined in the following subsection. It is also possible that, as a result of the functional forms of the model equations, nonunique parameter values may exist for certain types of data. Examples of this situation will be given in Secs. 5 and 6. The minimum amount of data required to completely specify the eddy model is n_V+1 measurements at different depths (at the same horizontal position and time) and n_H measurements at different horizontal locations and/or times.

b. Accurate parameter specification

To obtain a more accurate analytical approximation to an observed eddy, it is very desirable to utilize more than the minimum required data set, since no model can be expected to give an exact fit to the observed process. In addition, some of the inaccuracies in ocean measurements can be decreased by employing additional measurements.

The problem of generating more accurate choices of model parameters can be viewed as overspecifying systems (11) and

(14) by using more points (and hence more equations) than parameters. As discussed previously, it is appropriate to first choose the n_V vertical parameters, and then the n_H horizontal parameters. If horizontal position and time are fixed at (x_1, y_1, t_1) and observations $\Delta_1^{(j)}$ are obtained from $m_V (> n_V + 1)$ depths $z_1^{(j)}$, the vertical parameters can be varied to obtain a reasonable fit at all depths. The choice of these parameters could be made, for example, by minimizing the sum of the squares of error of the fit. However, we have found that, to avoid biasing the fit to the larger near-surface observations, a more accurate agreement with observed eddy structure is obtained by minimizing the sum of the relative squares of the error:

$$\min_{\underset{\sim}{p}_V, \bar{M}} \sum_{j=1}^{m_V} \left\{ [\bar{M} f_V(z_1^{(j)}, t_1; \underset{\sim}{p}_V) - \Delta_1^{(j)}]^2 / (\Delta_1^{(j)})^2 \right\}, \tag{15}$$

where $\bar{M} = f_H(x_1, y_1, t_1; \underset{\sim}{p}_H)$. We note that the magnitude of the horizontal structure $\bar{M}$ is varied since eliminating this quantity, using one of the measurements as in (11), can lead to significant errors if an inaccurate measurement is used for the elimination. Problem (15) can then be solved, for example by numerical techniques such as a modified Levenburg-Marquardt algorithm (Brown and Dennis, 1972), or by statistical nonlinear regression techniques, to minimize the error in interpolating between data points. This results in specifying the vertical structure. We note that for accurate specification of eddy vertical structure, we have found from our numerical experience that it is necessary to utilize readings to a depth of 750 m or more.

The choice of the n_H horizontal parameters requires m_H (> n_H) measurements at distinct horizontal positions and/or times. At each position and time (x_i, y_i, t_i), the average vertically-scaled deviation $\bar{\Delta}_i$ is computed as in (13). The resulting minimization problem to be solved in order to find the horizontal parameters is

$$\min_{p_H} \sum_{i=1}^{m_H} \left\{ [\bar{\Delta}_i - f_H(x_i, y_i, t_i; \underset{\sim}{p}_H)]^2 / (\bar{\Delta}_i)^2 \right\}, \tag{16}$$

where relative least squares is again advantageous to avoid biasing larger terms. We note that more accurate parameter fits will be obtained by using a wide distribution of points throughout the eddy, rather than a narrow grouping of points in one area, such as near the eddy edge. Further, because of the ordinarily large length scales of eddies, the points should be separated by at least 10 km to insure distinct readings; equivalently, if data is taken at the same horizontal location, but at different times, intervals of several days between points used in (16) is sufficient, assuming typical eddy drift speeds of 3-5 km/day.

Thus, knowledge of the minimum required data facilitates overspecification of the problems for the vertical and horizontal parameters. Such overspecification reduces the effects of errors in measurements and deficiencies in the model to give a more accurate analytical approximation to the eddy structure. If the environmental effects of an eddy are first observed at $m_V (>n_V+1)$ depths at the same horizontal position and time, the n_V vertical parameters, and hence the eddy vertical structure, may be specified by solving (15). Then, given m_H $(>n_H)$ measurements of environmental deviations at distinct horizontal positions and/or times, the remaining n_H horizontal parameters can be determined by solving (16). Thus, the eddy structure is specified at all locations and, if the original data is taken at distinct times, at all times.

c. <u>Specialization to one eddy model</u>

We conclude this section by specializing the discussion of previous subsections to the authors' model of Sec. 2. The parameters of this eddy model are the direction of rotation (specified by the sign in (7b)), depth of influence z_o, radius r_o, maximum rotational current speed U_o, and n_D parameters for the form assumed for the eddy trajectory. For example, if a constant drift velocity is assumed, the position of the eddy center at time t is given by

$$[x_o(t),\ y_o(t)] = [X_o + u_D t,\ Y_o + v_D t] \ , \qquad (17)$$

where (X_o, Y_o) is the position of the eddy center at time $t = 0$, and $\underset{\sim}{V}_D = (u_D, v_D)$ is the horizontal drift velocity. Thus, there are $n_D = 4$ drift parameters in this case. Simpler or more general forms for the drift trajectory may be assumed, but $n_D \geq 2$ since two parameters representing the initial position of the eddy center will always be present. We observe that the direction of rotation can be chosen unambiguously from the sign of the perturbation temperature, so that there are $n_H = n_D+2$ horizontal parameters. Moreover, $n_V = 1$ since the only vertical parameter is z_o. For any parameter values, deviations in pressure and density structure, and current velocities, are obtained from (5), (6), and (7) as

$$\bar{P} = (2\rho_o\Omega\sin\phi)U_o r_o K_1[J_o(\alpha_1 r/r_o) - J_o(\alpha_1)][F(z) - F(z_o)] \ , \quad (18a)$$

$$\bar{\rho} = (2\rho_o\Omega g^{-1}\sin\phi)U_o r_o K_1[J_o(\alpha_1 r/r_o) - J_o(\alpha_1)]F'(z), \quad (18b)$$

and

$$(u,v) = U_o K_1 \alpha_1 J_1(\alpha_1 r/r_o)[F(z)-F(z_o)][-y+y_o(t),x-x_o(t)]/r. \quad (18c)$$

The resulting temperature is then obtained from (1)-(3). Thus, this model is separable in the sense of subsection a.

The sole vertical parameter z_o may first be determined by measuring environmental deviations at a fixed horizontal position and time and at m_V (>2) different depths, solving problem (15) with $p_V^{(1)} = z_o$, and selecting the appropriate vertical function from (8). The horizontal parameters can then be obtained using m_H ($>n_H$) readings at distinct horizontal locations or times, and then solving (16) with the appropriate parameters and horizontal structure from (18). We shall illustrate this procedure in following sections.

4. Typical observational data

We consider here the use of temperature and current measurements to select model parameters. Our purpose is to consider the efficient use of such data, in the sense that problems (15) and (16) are numerically stable and that all the information implicit in the data is utilized.

a. Temperature measurements

The most commonly-observed environmental effect of oceanic eddies is the large temperature perturbations induced by their strong rotational currents. Each eddy produces a characteristic temperature perturbation, a fact which has been used in the tracking of eddies over extended periods of time (see, for example, Parker, 1971). Consequently, it is of obvious importance to consider specification of model parameters using temperature observations. In the process of constructing an eddy model, however, temperature is typically not a primary quantity. Solutions to the fluid equations are usually in terms of pressure, density, and current. Resulting temperature effects can then be determined by using a state equation such as (3), or tabulated values of sea water properties. Unfortunately, this determination is typically numerically unstable, in that small changes of perturbation density lead to large temperature variations, making accurate selection of parameters difficult. To avoid this problem, we propose preprocessing temperature measurements so that they may be used effectively in parameter selection.

Eddy temperature effects result from perturbations in the static density, salinity, and pressure profiles. By assuming small perturbations in these quantities from static values as in (1), and expanding (3) in the perturbed quantities, it can be shown that the resulting temperature perturbations are primarily functions of the perturbation density. Physically, this results from the fact that eddy-induced salinity variations are relatively small (Fuglister, 1972), and that pressure perturbations are small in comparison to the large hydrostatic pressure. We may then approximate the density by

$$\rho \doteq E[T, P_s(z)] \ , \tag{19}$$

where (19) represents either a form of state equation (3) with constant salinity or else a tabulated density value, and $P_s(z)$ is the static pressure in (1b). We note that $E(T,P)$ is a well-conditioned function of temperature, in that small errors in temperature will cause smaller errors in density. Moreover, the induced error in approximating density in (19) can be shown to be less than one percent of the eddy-induced perturbation density.

The static state of an ocean area under consideration is typically known from long-term observations or archival sources. Eddy density perturbations $\Delta\rho$ may then be approximated from the observed temperature perturbations ΔT by the relation

$$\Delta\rho = E(T_s + \Delta T, P_s) - E(T_s, P_s) \ , \tag{20}$$

where T_s is either the known static temperature or the theoretical static temperature derived from (1) - (3). Equation (20) gives perturbation density sufficiently accurate for selection of model parameters. Theoretical density predictions from the eddy model can then be used in comparison with those of (20) in the procedure of Sec. 3.

b. Current measurements

In recent years, current measurements have become increasingly important in studying large-scale oceanographic phenomena (see, for example, Treshinikov et al, 1977). Clearly, significant information about eddies can be acquired through the use of current meters (Koslyakov and Grachev, 1973).

A single current meter provides both the magnitude $|\underset{\sim}{V}|$ and the direction θ of the observed current, where we take the angle θ to be measured positively counterclockwise from an eastward latitudinal parallel. As with density measurements, observations of current magnitude can be used with predictions of current speeds from the model equations directly in the procedure of Sec. 3. On the other hand, current-direction readings provide additional information about the position of the eddy center, i.e., $x_o(t)$ and $y_o(t)$. How this information is exploited depends on the eddy model. In one with assumed circular streamlines, such as the model of Sec. 2, the unit tangent vector $(\cos\theta, \sin\theta)$ to a streamline at a given point will be perpendicular to a line through the eddy center. Thus, if a current direction θ_i is observed at the horizontal position (x_i, y_i) at time t_i, the eddy center is known to lie on the line

$$[y_o(t_i)-y_i] = [x_o(t_i)-x_i]\tan(\pi/2-\theta_i). \qquad (21)$$

Equation (21) is a relation between the coordinates $(x_o(t_i), y_o(t_i))$ of the eddy center at the time of the measurement. If current direction is measured at two or more points simultaneously, and if the points do not lie on the same eddy diameter, then $x_o(t_i)$ and $y_o(t_i)$ are uniquely determined by the intersection of lines in the $x_o y_o$ - plane of the form (21). Of course, this scheme is prone to serious numerical errors if the eddy is not nearly circular, if the constructed lines are nearly parallel, or if significant errors are present in the current measurements. Effects of the last two errors can be avoided by using several widely-spaced current meters, and avoiding readings where $|\underset{\sim}{V}|$ is small. In practice, measurements of currents would be discarded if the current speed is less than some minimum speed V_{min}, where V_{min} is sufficiently large (perhaps 20 cm/sec) that contributions from eddy drift or other environmental effects are insignificant.

5. Time-independent problems

Large numbers of eddy observations consist of temperature measurements, taken over sufficiently short time intervals so as to be considered time-independent. Several eddy experiments have made use of ship-dropped bathythermographs (BTs) to get continuous temperature readings to depths below that of significant eddy influence. For example, much data from the MODE, POLYGON, and POLYMODE experiments (see, for example, Volkmann, 1977) in the form of temperature sections taken in linear traversals of an ocean area, often obtained from expendable bathythermographs (XBTs), is available. XBTs are less expensive and easier to use than BTs but are restricted to depths of less than 2 km. Also, sections are obtained from airborne expendable bathythermographs (AXBTs), which are convenient and require lower support costs. Although they are restricted to depths of under 400 m at present, the development of deeper AXBTs is currently being considered.

We divide this section into two analytically similar, but experimentally distinct, parts. First, we consider data from a vertical planar cross section through an eddy, as might be obtained by a ship linearly traversing an eddy or by a plane dropping AXBTs along a linear trajectory. Then, data taken throughout the eddy in a nonplanar fashion is analyzed. The procedure of Secs. 3 and 4 and the model of Sec. 2 are used to approximate the corresponding environmental effects of such eddies.

In both the planar and nonplanar cases, the data is assumed to be taken over sufficiently short time intervals that eddy drift may be neglected. Thus, the coordinates of the eddy center, $x_o(t) = X_o$, $y_o(t) = Y_o$, are constant. These, together with the eddy radius r_o and maximum current speed U_o, result in four horizontal parameters (n_H=4), with eddy depth of influence z_o as the sole vertical parameter ($n_V = 1$). The direction of rotation is chosen immediately by the sign of the perturbation temperature.

a. Planar temperature data

Sampling of oceanic temperature may be obtained from ship-dropped BTs or XBTs or plane-dropped AXBTs along straight-line traversals of large ocean areas. Occasionally, large cold- or warm-temperature anomalies are observed, whose structure suggests the presence of a mesoscale eddy. Discrete drops along an eddy chord are illustrated by the crosses in Fig. 1a. A portion of an XBT section through the Atlantic, taken by Seaver (1975) as part of the MODE program, is shown in Fig. 2, indicating the presence of a large cyclonic eddy.

As proposed in Sec. 3, z_o is selected first, using temperature measurements at the same horizontal position (x_1, y_1) at m_V (>2) depths $z_1^{(j)}$, $1 \leq j \leq m_V$. Perturbation density $\Delta\rho_1^{(j)}$ is calculated at each depth as in (20), and is used as the environmental deviation in (15), with the theoretical perturbation density obtained from (18b). The solution of the resulting problem

$$\underset{z_o, \bar{M}}{\text{Min}} \quad \sum_{j=1}^{m_V} \left\{ [\Delta\rho_1^{(j)} - \bar{M}\, F'(z_1^{(j)})]^2 / (\Delta\rho_1^{(j)})^2 \right\} \tag{22}$$

then specifies the vertical eddy structure through (7c) and (18). At each of the m_H (>4) horizontal positions (x_i, y_i), and depths $z_i^{(j)}$, perturbation density is calculated as in (20). Then the average vertically-scaled perturbation density is computed from (13) and (18) as

$$\overline{\Delta\rho_i} = \sum_{j=1}^{m_V} [\Delta\rho_i^{(j)} / F'(z^{(j)})] / m_V \; . \tag{23}$$

Determination of the remaining horizontal parameters follows from (16), using $\overline{\Delta\rho}_i$ from (23) as the average vertically-scaled deviation, resulting in

$$\underset{U_o, r_o, X_o, Y_o}{\text{Min}} \sum_{i=1}^{m_H} \left\{ \overline{\Delta\rho}_i - (2\rho_o \Omega g^{-1} \sin\phi) U_o r_o K_1 [J_o(\alpha_1 r_i / r_o) - J_o(\alpha_1)]^2 / (\overline{\Delta\rho}_i)^2 \right\} , \tag{24a}$$

where

$$r_i = [(x_i - X_o)^2 + (y_i - Y_o)^2]^{1/2} . \tag{24b}$$

Unique determination of the eddy center is not possible here. As indicated in Fig. 1(a), two circular eddies can exist which possess the same temperature structure in a common vertical plane. One eddy is represented by the solid circle, whose center (X_o, Y_o) is a distance d from the chord. The second, indicated by the dashed circle, is of identical size and strength, but is centered at $(\bar{X}_o, \bar{Y}_o)$ at a distance d on the opposite side of the chord. Although differentiation between the two centers is not possible, values for the other horizontal parameters, including eddy size r_o and strength U_o, can be obtained. Any convergent scheme used in solving the minimization problems will estimate one of the two centers, and the position of the alternate center can be found by reflection across the eddy chord. The results of the planar temperature data problem are summarized in the first line of Table 1. In this example, eddy depth of influence z_o, radius r_o, and maximum current speed U_o can be uniquely determined. An ambiguity exists in the time-independent position of the eddy center, and no information is available concerning the drift of the eddy.

To illustrate our procedure, we use data from Fig. 2, taken every 20 km. At each horizontal location, the temperature was noted at seven equally-spaced depths from 150 m to 750 m. Solving (22) numerically gave a depth of influence of $z_o = 2640$ m. Eddy size and strength were determined by solving (24) numerically, resulting in $r_o = 150$ km and $U_o = 110$ cm/sec. The distance of closest approach of the chord to the eddy center was found to be 10 km, so that the ambiguity in the center location is small in this example. For an acceptable horizontal fit, at least eight readings were necessary to obtain physically reasonable parameter values. Addition of more points led to better agreement in the shape and elevation of the isotherms, although inclusion of more than twenty points led to insignificant changes in parameter values. An example of the dependence of parameter values on the amount of data will be presented in the following subsection. Resulting isotherms, obtained from (18), the ocean static state, and the inverted Eckart equation, are shown in Fig. 3. Satisfactory agreement is noted between depths and overall shapes of the observed and predicted isotherms in Figs. 2 and 3. Distributions of other eddy characteristics, such as currents, can be determined similarly.

b. <u>Non-planar temperature data</u>

We consider now the use here of certain non-planar temperature data from BTs, XBTs, or AXBTs. Such data is more useful than planar measurements in experiments designed to study a particular oceanic eddy. Many previous experimental studies have required deep temperature data to determine the full vertical structure and depths of significant eddy influence. However, if high accuracy at large depths is not required, then shallower data can be used to obtain the model approximation to the vertical eddy structure. With the model of Sec. 2, we shall focus on using AXBTs, although the results will be valid also for BTs or XBTs.

To determine the vertical structure, we have noted that measurements to depths of 750 m are necessary. As mentioned previously, although limited to depths of about 350 m at present, the possible development of deeper AXBTs is under investigation. Thus, estimation of z_o by solving (22) would require the use of one such AXBT, or another form of measurement of the eddy vertical structure.

With the eddy vertical structure specified, AXBTs are dropped at m_H (>4) horizontal positions (x_i, y_i), and readings are obtained at depths $z_i^{(j)}$. We illustrate in Fig. 1b one scheme for obtaining nonplanar data. The scheme was chosen for its simplicity, with no attempt being made here to select an optimum procedure, and presupposes a nearly circular eddy. A linear path L_1, with relatively widely spaced AXBT drops, for example every 25 km, is followed until several temperature anomalies are found, indicating the presence of a large eddy. Then, readings close to the eddy center are obtained by making more closely spaced drops, say every 10 km, following a path L_2 which is perpendicular to L_1 and passes through the position of maximum perturbed temperature obtained previously along L_1. The second chord then approximates a diameter. Information from both paths permits an accurate determination of U_o, r_o, X_o, and Y_o, by first computing the average vertically-scaled perturbation density as in (20) and (23) at each drop and then solving problem (24). Moreover, ambiguity in eddy center position, found in the previous subsection, no longer exists. Solution of the horizontal problem does not require data below 350 m, and hence AXBTs are ideal in providing data throughout the eddy, after the vertical structure is specified. Thus, in the case of nonplanar temperature data, unique determination of the time-independent parameters is possible, using one deep measurement at a fixed horizontal position. We summarize these results in the second line of Table 1. Here, eddy depth of influence, radius, maximum current speed, and position of the eddy center can be determined at the time of the measurements. However, nothing can be infered concerning the translation direction or speed of the eddy.

As an illustrative example, we use BT data from a large cyclonic eddy studied by Khedouri and Gemill (1974). The eddy was observed to have significant environmental effects to depths of over 2 km, with a radius of approximately 150 km, and a maximum current of 110 cm/sec. The observed temperature data in a vertical cross section through the eddy center was rotated to generate an axisymmetric eddy. A single temperature measurement (to 800 m) near the eddy center was used to determine the depth of influence z_o. The resulting value of 2600 m is in agreement with observations of the vertical extent of the eddy. The equivalent of AXBT data can be constructed by sampling the data only to a depth of about 350 m. Examples of typical temperature deviations at several distances from the eddy center are shown in Fig. 4(a). The temperature perturbations increase rapidly with depth, reaching maxima near 500 m (which of course is not observed using AXBTs). A small surface expression of the eddy, seen in the observational data, and presumably due to near-surface mixing, is not included in our model of Sec. 2.

Using the scheme of Fig. 1(b), twenty-three AXBTs were simulated, with readings taken at depths of 150 m, 250 m, and 350 m, at the positions illustrated in Fig. 5(a). We obtained from the solution of (24) the values U_o = 120 cm/sec, and r_o = 150 km. In addition, the eddy center was located within 6 km of the observed position. Thus, good agreement between theory and observations was obtained. Theoretical temperature perturbations are shown in Fig. 4(b). They exhibit features and magnitudes quite similar to those observed in the shallow depths of Fig. 4(a), with decay at larger depths similar to that observed in the BT readings. In addition, the model specifies all eddy characteristics at all locations. For example, Fig. 6 shows curves of constant rotational current speed, obtained from (18c), in a plane through the eddy center. The currents are small at all depths near the eddy edge and center, and current speeds decay rapidly with increasing depth from the surface maximum at all horizontal positions.

The number of points necessary for accurate parameter specification was tested by solving the horizontal problem (24) using data sets of increasing size taken from the eddy of Fig. 4(a).

Initially, three AXBT drops were made on L1 (Fig. 5(a)) and two on L2, and the resulting horizontal problem solved. Additional AXBT drops were made, until all twenty-three AXBTs were included. After each drop, U_o, r_o, x_o and y_o were determined by solving (24). Resulting values of U_o and r_o are plotted in Fig. 5(b) as a function of the number of AXBTs utilized in the horizontal fit. In this example, utilization of as few as eight AXBTs led to good estimates of parameter values. However, variations of as much as 10% in parameter values from the "final" values of U_o = 120 cm/sec and r_o = 150 km are noted if fewer than twenty AXBTs are utilized. Similar results are observed in the location of the eddy center. Of course, the accuracy of the parameter specifications varies not only with the number of AXBTs used, but also with the positions of the AXBTs, the eddy under consideration, and the accuracy of the model approximation. We have conducted similar studies on other observed eddies and with other horizontal locations of the readings. From these studies, we have found that a minimum of eight AXBTs, dropped near both the eddy edge and center, are ordinarily necessary for good estimates of parameter values. Variation of parameter values decreased with increasing density of AXBTs, until inclusion of more than twenty AXBTs led to insignificant parameter variation, as in Fig. 5(b).

6. Time-dependent problems

We consider here the analysis of eddy observations from fixed moorings, containing instrument packages at discrete fixed depths. In contrast to BTs, the time series of data from moorings can provide information on eddy translational rates and directions, as well as on eddy size and strength. Temperature observations are available from thermistor strings, while current-temperature moorings provide both temperature and current data; we will discuss each of these separately in Secs. 6(a) and 6(b), respectively, using the model of Sec. 2.

a. Thermistor-string data

We first consider the use of data from a single thermistor string, with thermistors located at m_V (>2) depths. At a fixed time t_1, eddy vertical structure may be determined first by solving problem (22). At each time t_i, the average vertically-scaled perturbation density $\Delta\rho_i$ is then determined by (23). To further describe the eddy, we would like to specify the translation of the eddy center $x_o(t)$ and $y_o(t)$. However, using a single thermistor string, it will be possible to determine only the speed by which the eddy drifts, not the direction of translation. That is, only the eddy drift speed,

$$S(t) \equiv |\underset{\sim}{V}_D(t)| = \{[x_o'(t)]^2 + [y_o'(t)]^2\}^{1/2} \quad , \qquad (25)$$

can be found. We shall assume here that the drift speed is constant, as in (17), so that

$$S = (u_D^2 + v_D^2)^{1/2} \qquad (26)$$

is one horizontal parameter in our problem. This assumption may be valid if the eddy is observed only over a sufficiently short time interval, or if the eddy is not significantly influenced by other ocean currents and by topographical features. Moreover, any eddy of a fixed strength and at the same radial distance r from the thermistor string will produce identical temperature effects. Hence, it is not possible to determine the exact initial position (X_o, Y_o) but only the initial distance R from the string. Thus

$$R = (X_o^2 + Y_o^2)^{1/2} \tag{27}$$

is a second horizontal parameter. Further, only the distance d of closest approach of the eddy center to the string can be determined, giving a third horizontal parameter. The eddy radius r_o and maximum current speed U_o comprise the remaining horizontal parameters, so that $n_H = 5$.

To determine the five horizontal parameters, the vertically-scaled perturbation density $\overline{\Delta\rho}_i$ is computed at m_H (>5) times t_i, $i = 1,\ldots,m_H$. To determine the predicted effects through (18b), the radial distance r must be computed at each time. We define a moving coordinate system (x',y'), with origin at the eddy center. The x' -axis is parallel to the direction of eddy drift, with decreasing x' in the direction of V_D, as illustrated in Fig. 7(a). To an observer fixed in the (x',y') system, the thermistor string will appear to translate in the positive x'-direction with speed S. Initially, the eddy center will be located at

$$(X_o', Y_o') = [-(R^2-d^2)^{1/2}, -d] \ . \tag{28a}$$

From elementary geometry, the desired radial distance $r(t)$ at any time t will then be

$$r(t) = [(X_o'+St)^2 + d^2]^{1/2} \ . \tag{28b}$$

Equations (28) together with (24), comprise the problem for the five horizontal parameters. Examination of the problem shows that it can be solved only for the parameter combinations $U_o r_o, R/r_o, d/r_o$, and S/r_o. That is, unique solutions for individual parameter values are not possible, since U_o, R, d, and S can be determined only as (simple) functions of r_o. Thus, each member of a one-parameter family of eddies is a possible fit to the observed temperature-perturbation data. The nonunique parameter specification is summarized in the third line of Table 1. Eddy depth of influence can be determined uniquely, but eddy radius, maximum current speed, and the eddy center position and trajectory can only be determined to within a one-parameter family of values, assuming a linear drift form.

To illustrate this nonuniqueness, we present simulated thermistor string data in Fig. 8 for a large cyclonic eddy with linear drift. The data for the resulting cold-core eddy was generated using the model of Sec. 2, assuming a drift speed $S = 5$ km/day, radius $r_o = 125$ km, maximum current speed $U_o = 150$ cm/sec, closest approach distance $d = 20$ km, and depth of influence

$z_o = 2100$ m. We note that, at any depth, temperature decreases as the eddy approaches the thermistor string, and then increases to the static state as the eddy recedes. At the initial time $t = 0$, the thermistor was chosen to be on the eddy edge, so that $R = r_o$. Applying the procedure of this section, readings taken at the depths shown on Fig. 8 every five days reproduce the exact (inputed) value of $z_o = 2100$ m, and yield constant values for $U_o r_o$, d/r_o, and S/r_o. The parameters U_o, d, and S are illustrated as functions of r_o in Fig. 9. We note that for any assigned value of the eddy radius, S, d, and U_o are uniquely specified, as indicated by the dashed line corresponding to the exact radius $r_o = 125$ km. To generate curves such as those in Fig. 9, a minimum of four readings at different times is required; for accurate specification, we have found that at least eight readings are necessary, each separated by several days.

Addition of more than one thermistor string at different horizontal positions supplies sufficient information to determine unique parameter values, and to specify drift direction as well as speed. The vertical parameter z_o is again chosen first by using data from any one string at a fixed time. If we assume the linear trajectory (17), the radial distance $r_i(t)$ from the thermistor string T_i to the eddy center at time t is clearly

$$r_i(t) = \left\{[x_i - (X_o + u_D t)]^2 + [y_i - (Y_o + v_D t)]^2\right\}^{1/2} . \quad (29)$$

To determine the horizontal parameters U_o, r_o, X_o, Y_o, u_D, and v_D, m_H (>6) readings are required. Using measurements from at least two thermistor strings, a unique solution can be obtained, since parameters now occur in (24a) and (29) in the six distinct combinations $U_o r_o$, $1/r_o$, X_o/r_o, Y_o/r_o, u_D/r_o, and v_D/r_o. To accurately specify model parameters, a significant overlap in time (say 15 days) from data taken from different strings must be present, and separation between thermistor strings should not be small in comparison to the size of the eddy (for example, 50 km). Thus, by the inclusion of additional thermistor strings, the indeterminacy of the one string case is avoided, and unique parameter specification is possible. Results are summarized in the fourth line of Table 1. Eddy depth of influence, radius, maximum current speed, center position and drift, assuming linear drift, are uniquely determined. More complex forms for the eddy translation $x_o(t)$ and $y_o(t)$ may be assumed, leading to additional parameters, which can also be uniquely determined using two or more thermistor strings.

b. Temperature and current data

Current data provides significantly more information than is available from temperature measurements alone. The inclusion of current meters on a thermistor string provides additional information on both the eddy strength at a given time, through current speed observations, and the location of the eddy center, via current direction.

We first consider a single temperature-current mooring with m_c current meters at depths $\eta^{(j)}$ $1 \leq j \leq m_c$, and m_V thermistors at depths $z^{(\ell)}$, $1 \leq \ell \leq m_V$. At time t_i, the current direction and magnitude at the j^{th} meter are $\theta_i^{(j)}$ and $V_i^{(j)}$. The vertical structure can again be chosen first by using temperature readings at a fixed time t_i and solving problem (22). We note that current magnitude could also be used as the environmental deviation in such a determination, in which case the problem becomes

$$\underset{z_o,\bar{M}}{\text{Min}} \sum_{j=1}^{m_c} \left[V_i^{(j)} - \bar{M}[F(\eta^{(j)})-F(z_o)]\right]^2/(V_i^{(j)})^2 \qquad (30)$$

where F(z) is given by (7c). We caution again against using readings where current magnitudes are less than V_{min} (say 20 cm/sec), as errors induced by eddy drift and other environmental effects might become significant. Alternatively, both current and temperature data could be used in selecting z_o, minimizing the sum of the squares of both (22) and (30). At any time, the average vertically-scaled perturbation density can be determined and, similarly, the average vertically-scaled current speed $\bar{V}_i$ given as

$$\bar{V}_i=(1/m_c) \sum_{j=1}^{m_c} V_i^{(j)}/[F(\eta^{(j)})-F(z_o)] \qquad (31)$$

where readings with currents below $\bar{V}_{min}$ are not included.

Current direction measurements can then be used to simplify the horizontal problem. At any time t_i, the average direction $\bar{\theta}_i$ can be obtained by averaging the current direction from each meter for which $V_i^{(j)} > V_{min}$. The relationship (21) gives a line of possible centers, giving $y_o(t_i)$ in terms of $x_o(t_i)$, and hence eliminating the parameters implicit in $y_o(t)$. For the form of $x_o(t)$, we again assume a linear drift, (17), although a nonlinear form can be treated by introducing the appropriate parameters. Thus, there are four horizontal parameters, X_o, u_D, r_o, and U_o, and the sole vertical parameter z_o. From (21), the radial distance from the eddy center at time t_i is given by

$$r(t_i) = |X_o + u_D t_i| \; [1 + \tan^2(\pi/2 - \bar{\theta}_i)]^{1/2}. \qquad (32)$$

We note from Fig. 6 that, near the eddy center or edges, eddy currents at all depths are small. Hence the use of current direction to eliminate the parameters in $y_o(t)$, through (32), should be avoided in these regions. The problem for determining the remaining model parameters, given readings at m_H (>4) distinct times when significant currents ($>V_{min}$) are present is

$$\underset{X_o, u_D, r_o, U_o}{\text{Min}} \sum_{i=1}^{m_H} \left\{ \left(\overline{\Delta\rho_i} - (2\rho_o \Omega\, g^{-1} \sin\phi)\, U_o r_o\, K_1 [J_o(\alpha_1 r/r_o) - J_o(\alpha_1)] \right)^2 / (\overline{\Delta\rho_i})^2 + [\bar{V}_i - U_o\, K_1 \alpha_1 J_1(\alpha_1 r/r_o)]^2 / \bar{V}_i^2 \right\}. \qquad (33)$$

where $r(t_i)$ is obtained from (32). Since U_o now occurs distinctly in the current term, unique solutions can be found. This was not possible when only temperature data was available. Eddy depth of influence, radius, maximum current speed, center position and linear drift can all be determined uniquely, as summarized in line 5 of Table 1.

Suppose now that n (>1) current-temperature moorings are present at any time t. From the discussion of Sec. 4, the position of the eddy center $(x_o(t), y_o(t))$ can be determined by the intersection of perpendiculars to the average vertically-scaled current at each mooring computed as in (31). This is indicated in Fig. 7(b) for the case of two current meters. The position of the eddy center is known whenever two or more current meters measure significant currents; for accurate specification of the center position, readings should be taken from more than two moorings at several times. Of course, because of observational inaccuracies, if $n > 2$, the $(n-1)!$ intersection points would not coincide. They would be expected to be close enough so that a reasonable estimate of the center could be made, if the guidelines of Sec. 4 are followed; otherwise, the radially symettric model may not be applicable to the observed eddy. The vertical structure is specified next by using readings from a single mooring at a fixed time, and solving problem (22) or (30). Average vertically-scaled perturbations and current speeds are then computed at each mooring and time, and a problem similar to (33) solved for the remaining horizontal parameters U_o and r_o. Thus, with two or more current moorings, the eddy position at any time, as well as the size and strength, can be determined. This is summarized in the last line of Table 1. Eddy depth of influence, radius, maximum surface current speed, and eddy center position and drift can all be determined, independent of any assumptions concerning the form of the drift trajectory.

7. Summary

The major purpose of this paper is to describe the use of analytic models to determine approximate environmental effects of an oceanic eddy using limited observational data. The question of the amount and types of data necessary for such model specification is discussed, and techniques for accurate model parameter specification presented.

In order to provide illustrative examples of the ideas in this paper, a previously-derived model that gives predictions of eddy currents, density, pressure, and temperature is briefly reviewed. This model has eddy radius, maximum current speed, depth, and drift trajectory as parameters, but any model with an arbitrary number of parameters can be utilized. Thus, parameter specification for a general model is discussed. Separable models are considered, in which parameters associated with the vertical and horizontal structure are determined separately. The minimum amount of data required to specify an eddy model, a necessary prerequisite to accurate model approximation, is analyzed. Then, an efficient strategy is developed for accurate determination of model parameters, employing successive minimization problems for the vertical and horizontal parameters. These parameters and the equations specifying them are then described for the eddy model previously presented.

The use of oceanographic data in parameter-specification schemes is discussed. Although the direct use of temperature data can result in intolerable numerical errors, density anomalies induced by an eddy can be used to accurately determine model parameters. The use of current measurements are discussed also. The procedures are then applied to several typical experimental situations, using our particular eddy model as an example. First, temperature data acquired on a linear path through the eddy, as might be obtained during a single traversal of an ocean region, is considered. Assuming that the data are taken over small enough intervals to be time-independent, it is shown that the size and strength of a particular eddy can be determined uniquely. However, an ambiguity results in the position of the eddy center, so that either of two possible eddies could be responsible for the observed temperature perturbations. Next, we study non-planar time-independent data, with emphasis on that from AXBTs, as might be obtained during an extensive study of an eddy. It is shown that unique determination of eddy size, strength, and position is possible, although no information may be learned about the eddy drift. Examples are presented from actual eddy observations, with eddy size and strength determined to within five percent of observed values, and center position accurate to within 10 km. Guidelines are given for placement of bathythermographs for accurate parameter specification, and numerical sensitivity to the amount and position of data is considered.

Two distinct types of time-series data are examined, the first being only temperature readings that might be obtained from one or more thermistor strings, and the second including both temperature and current readings from one or more moorings. For a single thermistor string, it is shown that the model parameters cannot be determined uniquely, but that possibilities are restricted to a one-parameter family of eddies. This includes drift speed, but not direction, if a constant drift velocity is assumed. Addition of more strings, however, does lead to unique parameter values, as well as specification of drift direction. Addition of current meters to a single thermistor string leads to complete eddy specification when linear drift is assumed. Finally, data from two or more such moorings can be used to determine arbitrary drift as well as eddy depth, radius, and maximum rotational current speed.

The use of analytical eddy modeling simplifies the amounts and types of data required to determine eddy size, strength, and motion, and permits efficient approximating of eddy environmental effects. Future work in refining the parameter specification technique should be in the areas of model improvement and testing with other eddy models, further sensitivity analysis by comparison of predictions and observations using different and varied data sets and estimation procedures, and utilization of the technique with other forms of observations such as float paths. Hopefully, such refinements will contribute to the use of analytical modeling in describing ocean processes.

TABLE 1. Summary of parameter-specification results for various data types, using eddy model of Sec. 2.

Parameter / Data Type	Depth of Influence z_o	Eddy Radius r_o	Maximum Surface Current Speed U_o	Eddy Center $(x_o(t), y_o(t))$	Drift Speed $S(t)$
Planar Temperature Data (Fig. 1a)	U	U	U	A,I	
Non-planar Temperature Data (Fig. 1b)	U	U	U	U,I	
One Thermistor String (Fig. 7a)	U	F	F	F,L	F,L
Two or More Thermistor Strings	U	U	U	U,L	U,L
One Current-Temperature Mooring	U	U	U	U,L	U,L
Two or More Current-Temperature Moorings (Fig. 7b)	U	U	U	U	U

U=Uniquely Determined

A=Ambiguity Between Two Values

I=Time-independent Specification

F=Restricted to a Family of Parameters

L=Linear Drift Assumed

REFERENCES

Andrews, J.C., and P. Scully-Power, 1976: The structure of an east Australian current anticyclonic eddy. J. Phys. Oceanogr., 6, 756-765.

Baer, R.N., and M.J. Jacobson, 1974: Analysis of the effect of a Rossby wave on sound speed in the ocean. J. Acoust. Soc. Amer., 55, 1178-1189.

Bretherton, F.P., R.E. Davis, and C.B. Fandry, 1976: A technique for objective analysis and design of oceanographic experiments applied to MODE-73. Deep-Sea Res., 23, 559-582.

Brown, K.M., and J.E. Davis, 1972: Derivative free analogues of the Levenberg-Marquardt and Gauss algorithums for 43
nonlinear least squares approximations. Numerische Mathematik, 18, 289-297.

Bryden, H., G. Flierl, H. Freeland, N. Hogg, E. Katz, J. McWilliams, J. Richman, T. Sanford, J. Swallow, and C. Wunsch, 1975: Balance of Terms. Dynamics and the Analysis of MODE-1, Report of the MODE-1 Dynamics Group, Massachusetts Institute of Technology, 27-93.

Eckart, C., 1958: Properties of water part II: the equation of state of water and seawater at low temperatures and pressures. Am. J. Sci., 256, 225-240.

Flierl, G.R., 1977: The application of linear quasigeostrophic dynamics to Gulf Stream rings. J. Phys. Oceanogr., 7, 365-379.

Flierl, G.R., 1976: Contributions to theory and modelling of eddies. Theory and Modelling of Ocean Eddies: Contribution of the U.S. Delegation to the Yalta Polymode Theoretical Institute, Massachusetts Institute of Technology.

Freeland, H.J., and W.J. Gould, 1976: Objective analysis of meso-scale ocean circulation features. Deep-Sea Res., 23, 915-924.

Fuglister, F.C., 1972: Cyclonic rings formed by the Gulf Stream, 1965-66. Studies in Physical Oceanogr., Vol. 1, Gordon and Breach, 137-167.

Henrick, R.F., W.L. Siegmann, and M.J. Jacobson, 1977: General analysis of ocean eddy effects for sound transmission applications. J. Acoust. Soc. Amer., 62, 860-870.

Koshlyakov, M.N., and Y.M. Grachev, 1973: Meso-scale currents at a hydrophysical polygon in the tropical Atlantic. Deep-Sea Research, 20, 507-526.

Lai, D.Y., and P.L. Richardson, 1977: Distribution and Movement of Gulf Stream rings. J. Phys. Oceanogr., 7, 670-683.

Parker, C.E., 1971: Gulf Stream rings in the Sargasso Sea. Deep-Sea Research, 18, 981-993.

Seaver, G., 1975: Two XBT sections in the North Atlantic. MODE Hot Line News, WHOI 84:1 (unpublished document).

Stern, M.E., 1975: Minimal properties of planetary eddies. J. Marine Res., 33(1), 1-13.

Treshinikov, A.F., R.D. Pillsbury, W.D. Nowlin, Jr., E.I. Sarakhanyan and N.P. Smirnov, 1977: A comparison of summer current measurements in the Drake Passage. J. Phys. Oceanogr., 7, 610-614.

Volkmann, G., 1977: North Atlantic XBT sections. Atlas of the Mid-Ocean Dynamics Experiment (MODE-1), Massachusetts Institute of Technology, 69-90.

Watson, J.G., W.L. Siegmann, and M.J. Jacobson, 1976: Deep-ocean dynamics for environmental-acoustics models. J. Acoust. Soc. Amer., 55, 1178-1189.

Wiebe, P.H., E.J. Carpenter, A.E. Jahn, G.P. Knapp, III, S.H. Boyd, P.B. Ortner, and J.L. Cox, 1976: Gulf Stream cold core rings: Large-scale interaction sites for open ocean plankton communities. Deep-Sea Research, 23, 695-710.

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FIGURE LEGENDS

Fig. 1 (a) Chord through eddy, showing ambiguity in eddy center locations, (b) one scheme for obtaining non-planar eddy temperature data.

Fig. 2 Experimental isotherms (interpolated) through a large cyclonic eddy (adapted from Seaver, 1975).

Fig. 3 Theoretical isotherms for an eddy with radius 150 km, maximum current speed 110 cm/sec, and depth of influence 2640 m.

Fig. 4 Eddy temperature perturbations: (a) "observational" AXBT data, (b) theoretical results.

Fig. 5 (a) Position of AXBT drops in eddy of Fig. 4, (b) variation of eddy radius and maximum current speed with increasing numbers of AXBTs. 46

Fig. 6 Theoretical current structure through an eddy center with radius 147 km, maximum current speed 123 cm/sec, and depth of influence 2600 m.

Fig. 7 (a) One thermistor string geometry, (b) location of eddy center using two current meters.

Fig. 8 Model-simulated thermistor string data from eddy with linear drift speed 5 km/day, radius 125 km, maximum current speed 150 cm/sec, closest-approach distance 25 km, and depth of influence 2100 m.

Fig. 9 Family of parameter values for eddy of Fig. 7.

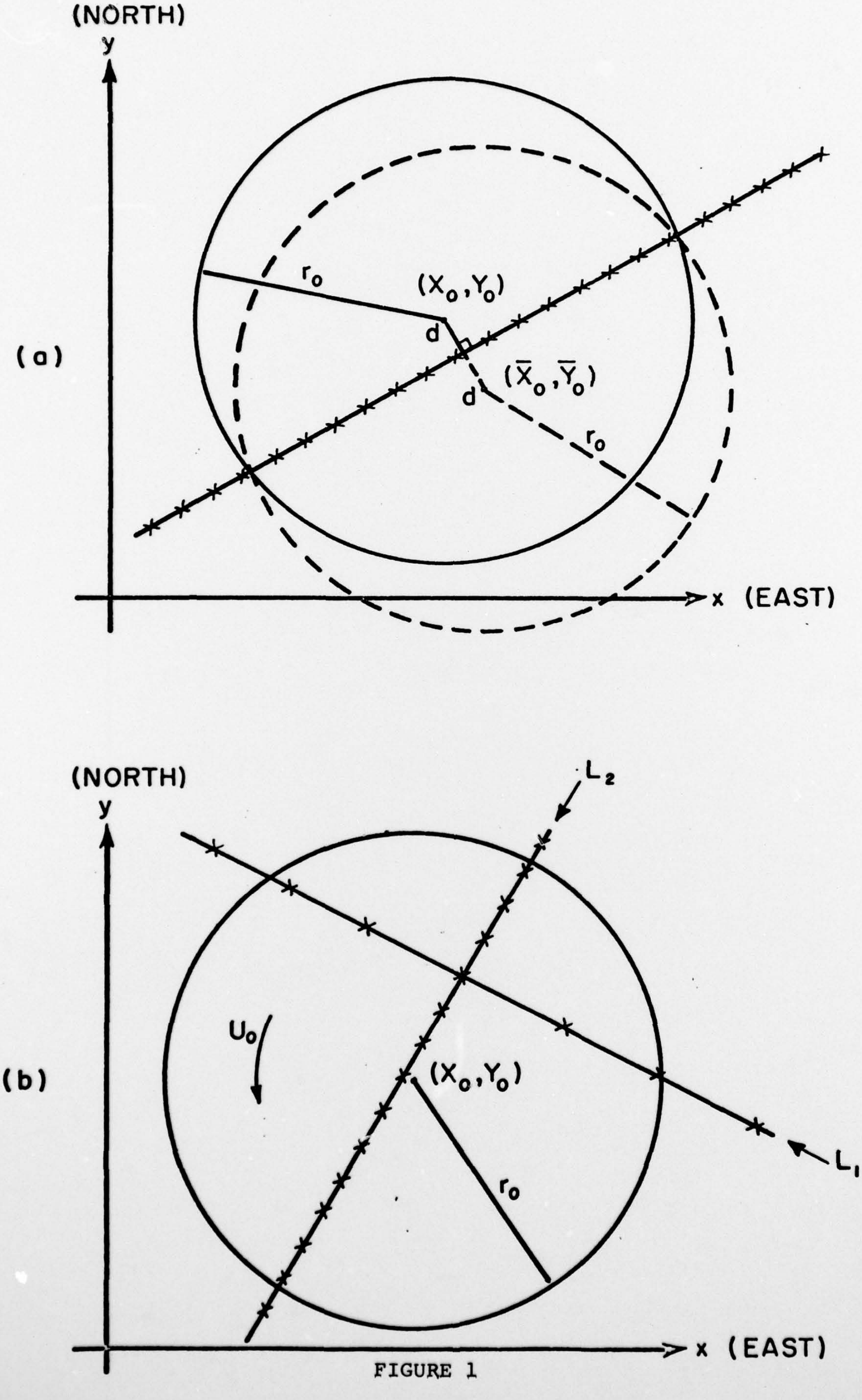
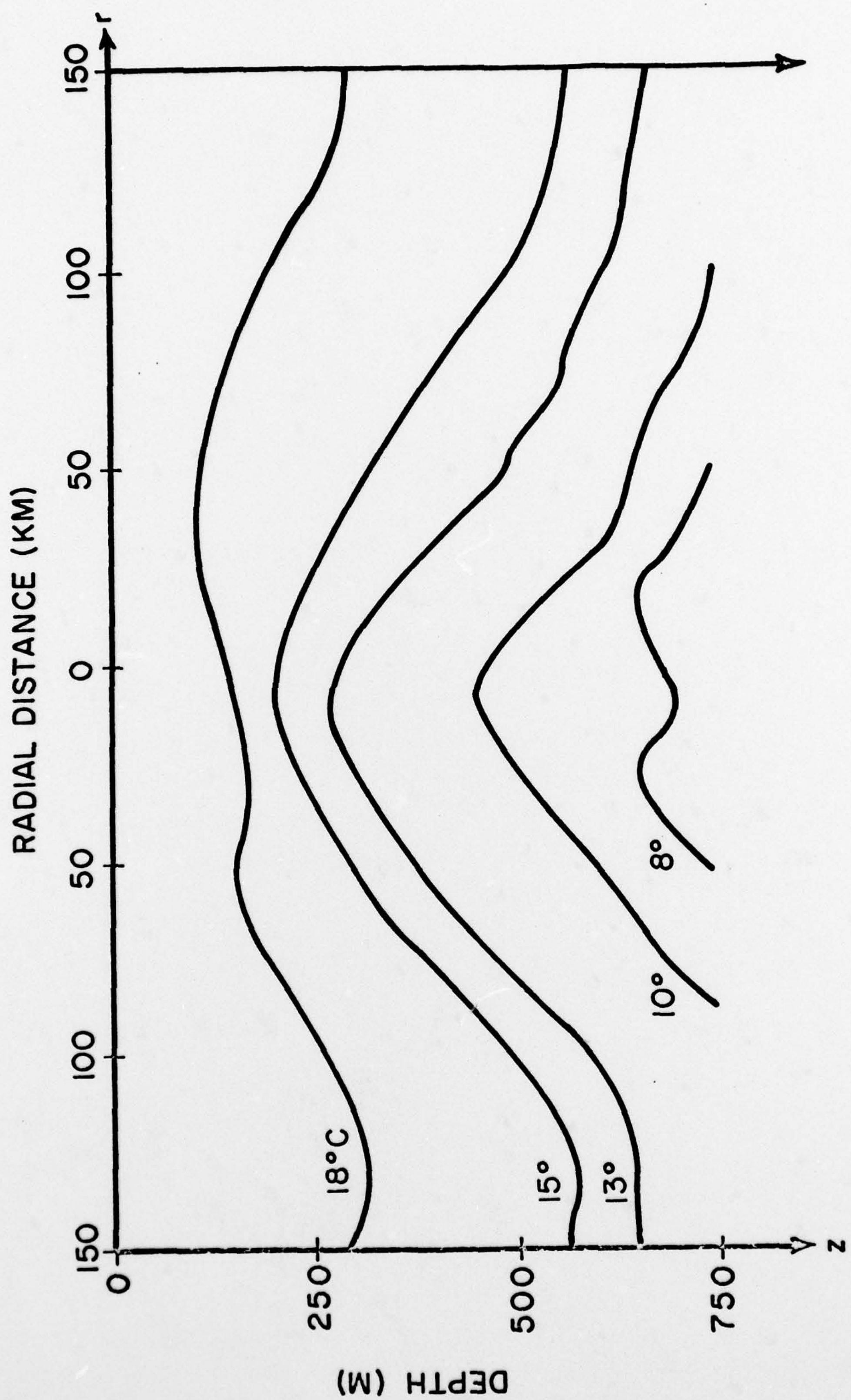

(NORTH)
y
r_o
(X_o, Y_o)
d
(a)
$(\bar{X}_o, \bar{Y}_o)$
d
r_o
x (EAST)
(NORTH)
y
L_2
U_o
(b)
(X_o, Y_o)
L_1
r_o
x (EAST)

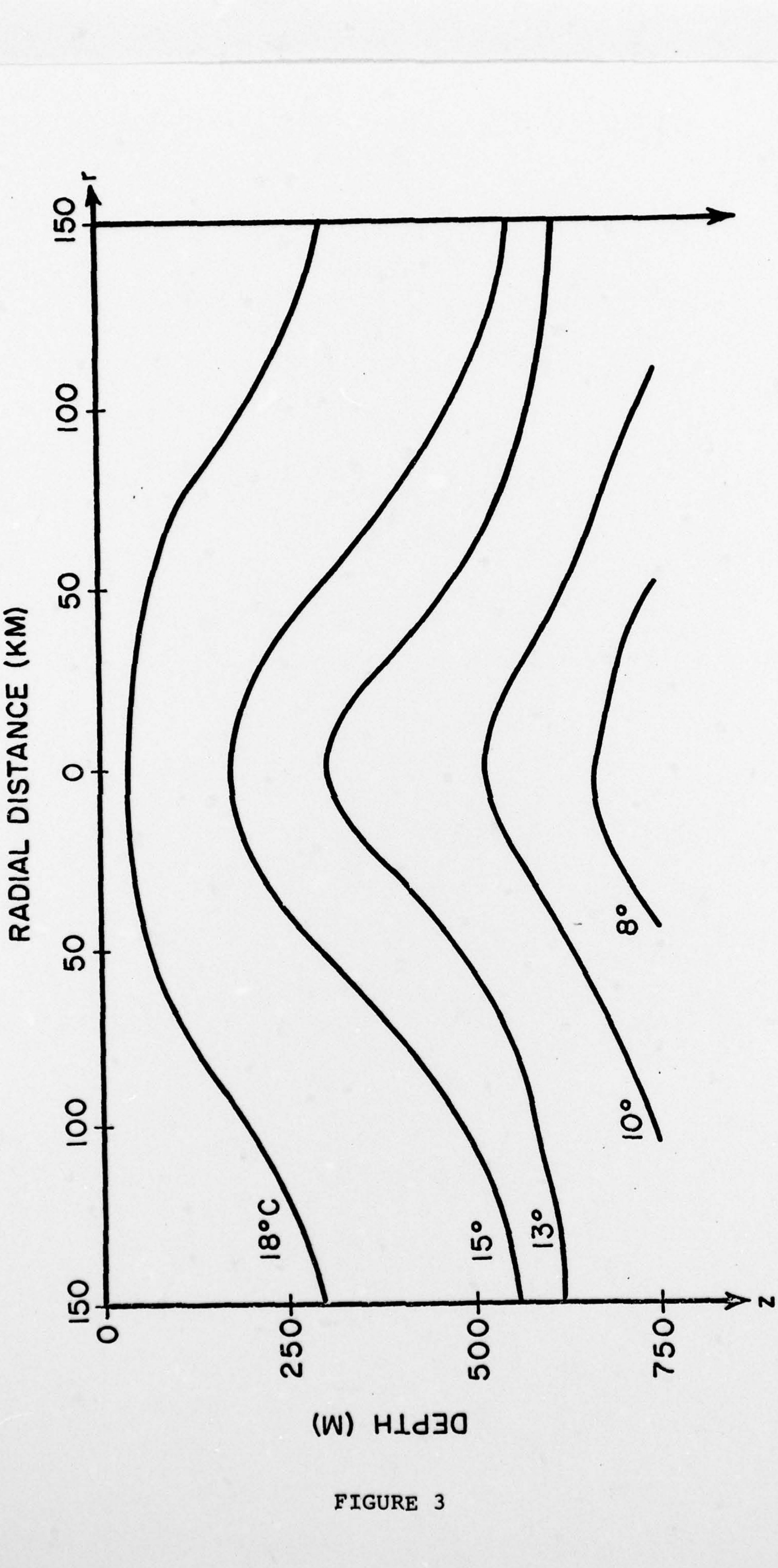
FIGURE 1

47


RADIAL DISTANCE (KM)
DEPTH (M)
150
100
50
0
50
100
150
r
z
0
250
500
750
18°C
15°
13°
10°
8°

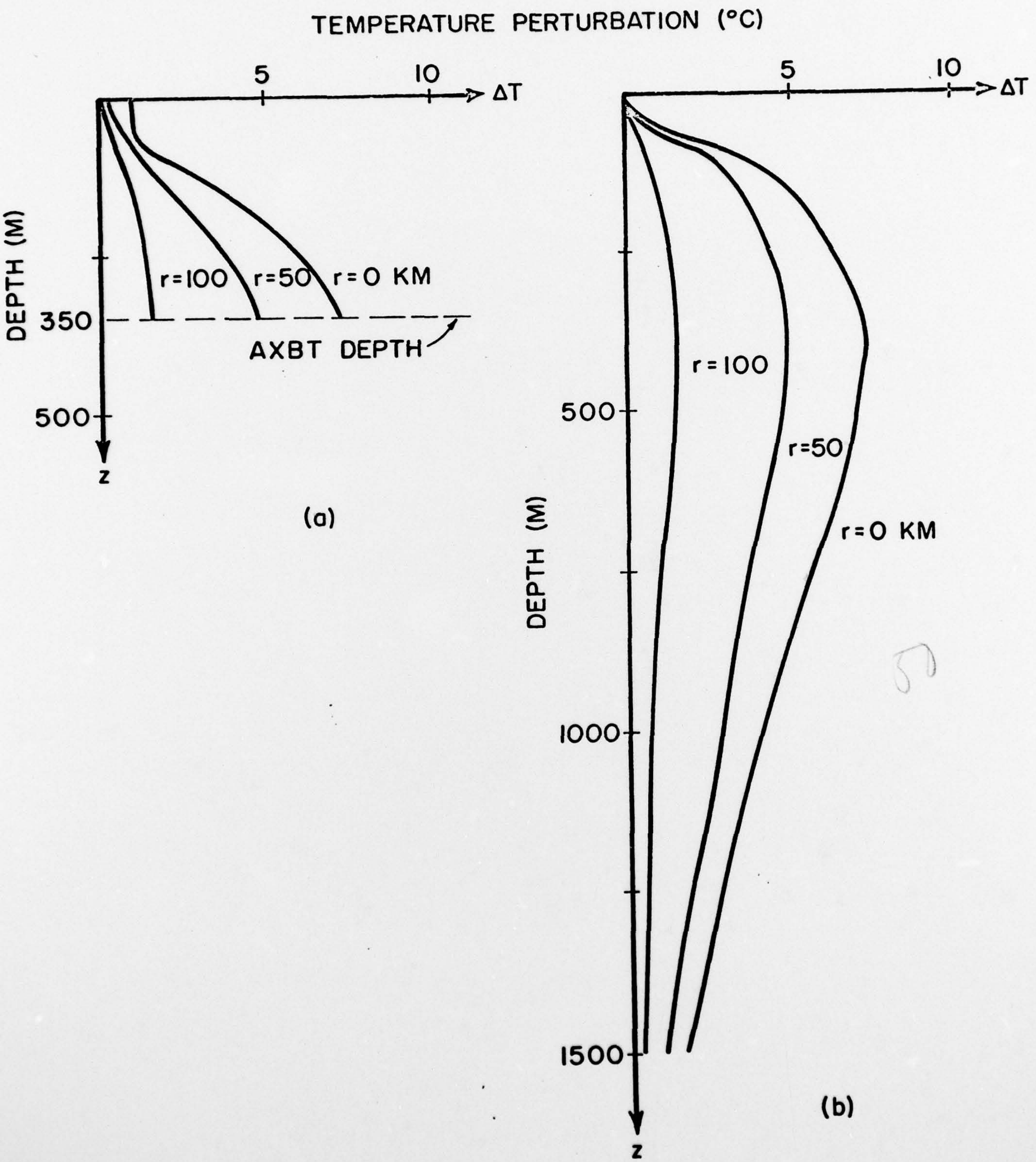
48

FIGURE 2


RADIAL DISTANCE (KM)
150
100
50
0
50
100
150
r
DEPTH (M)
0
250
500
750
z
18°C
15°
13°
10°
8°

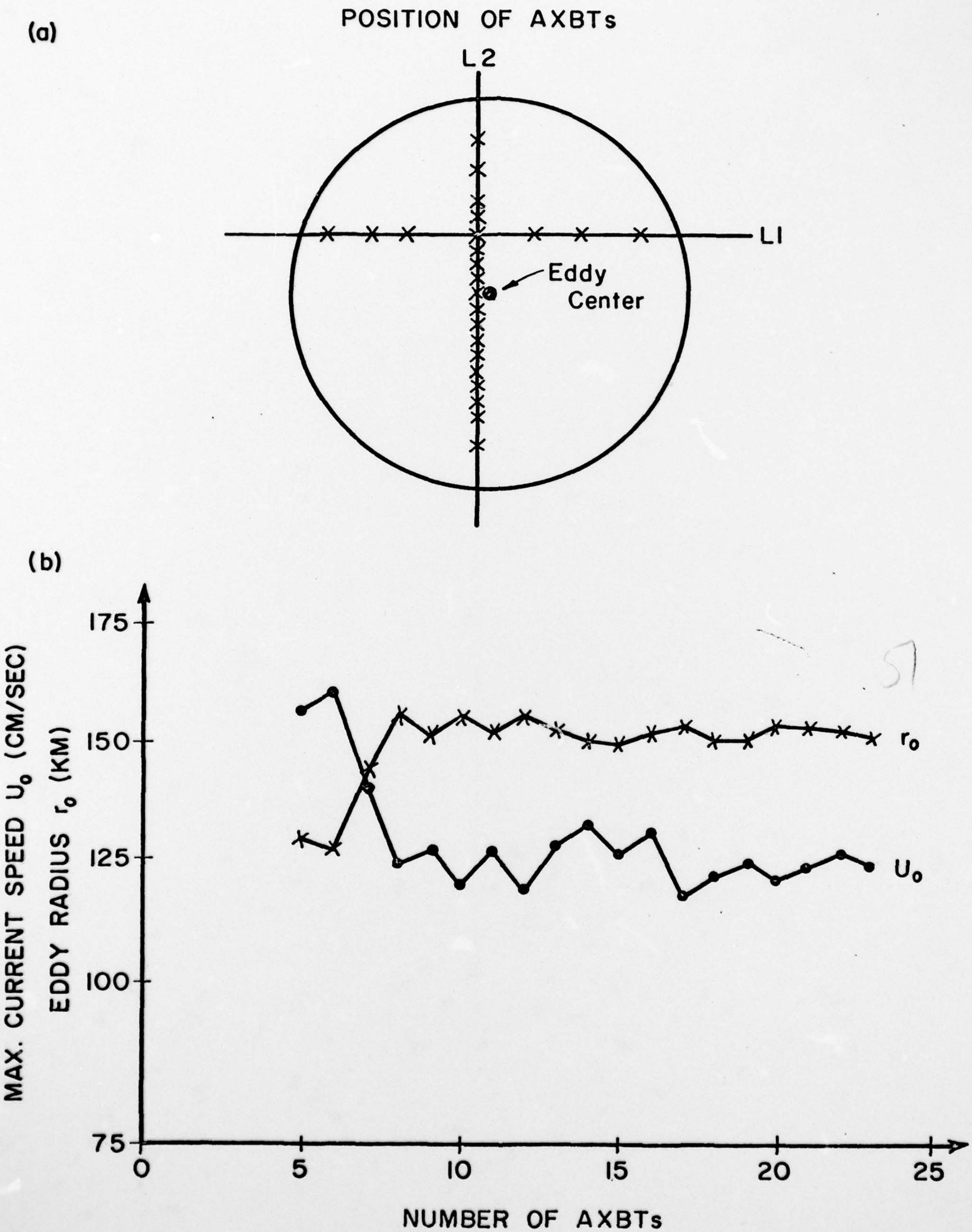
49

FIGURE 3



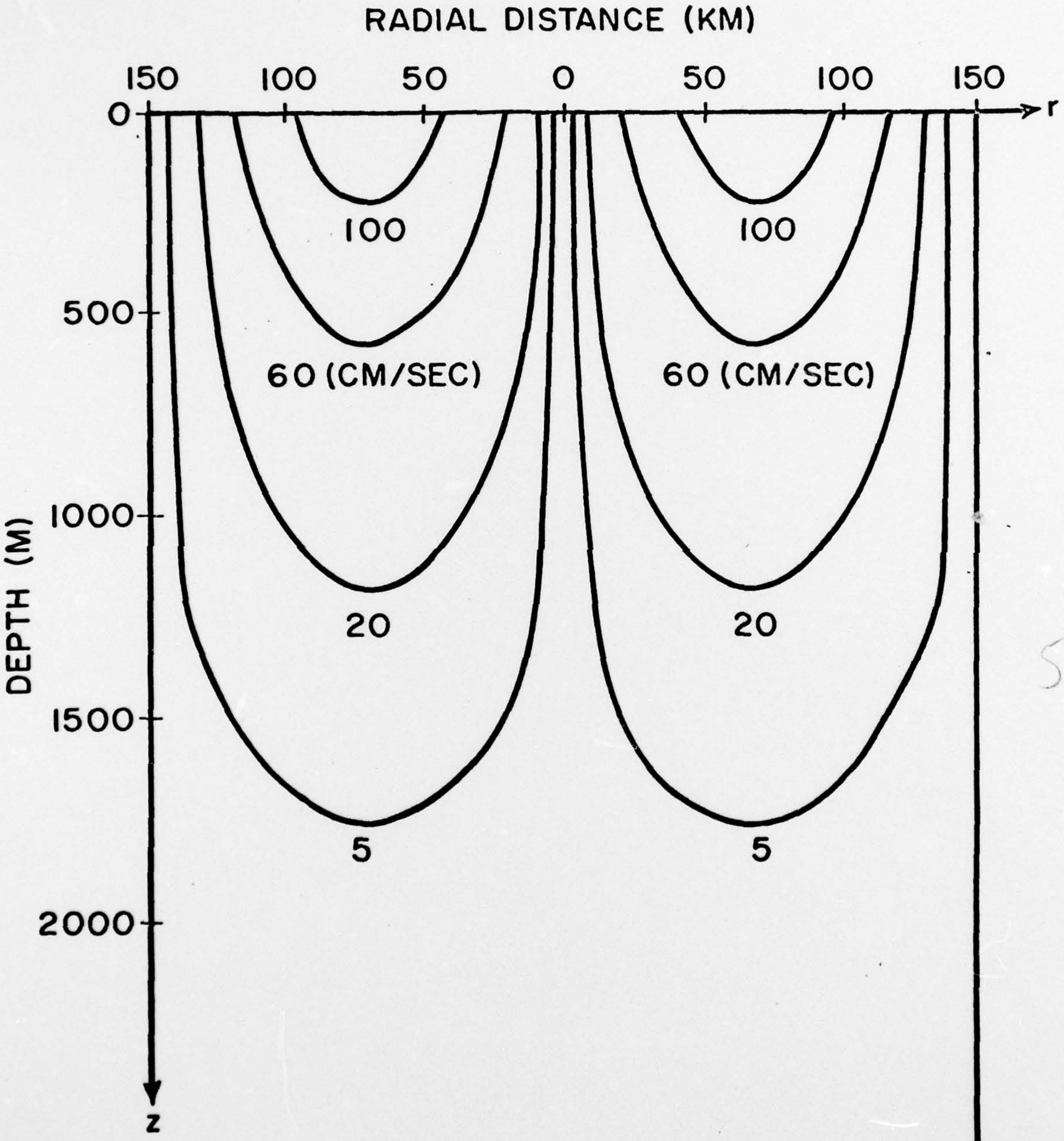
TEMPERATURE PERTURBATION (°C)
5
10
ΔT
DEPTH (M)
r=100
r=50
r=0 KM
350
AXBT DEPTH
500
z
(a)
5
10
ΔT
r=100
500
r=50
DEPTH (M)
r=0 KM
1000
1500
(b)
z

FIGURE 4



(a)
POSITION OF AXBTs
L2
L1
Eddy Center
(b)
MAX. CURRENT SPEED U_0 (CM/SEC)
EDDY RADIUS r_0 (KM)
175
150
125
100
75
0
5
10
15
20
25
r_0
U_0
NUMBER OF AXBTs

FIGURE 5



RADIAL DISTANCE (KM)
150
100
50
0
50
100
150
r
0
500
1000
1500
2000
DEPTH (M)
z
100
100
60 (CM/SEC)
60 (CM/SEC)
20
20
5
5

FIGURE 6

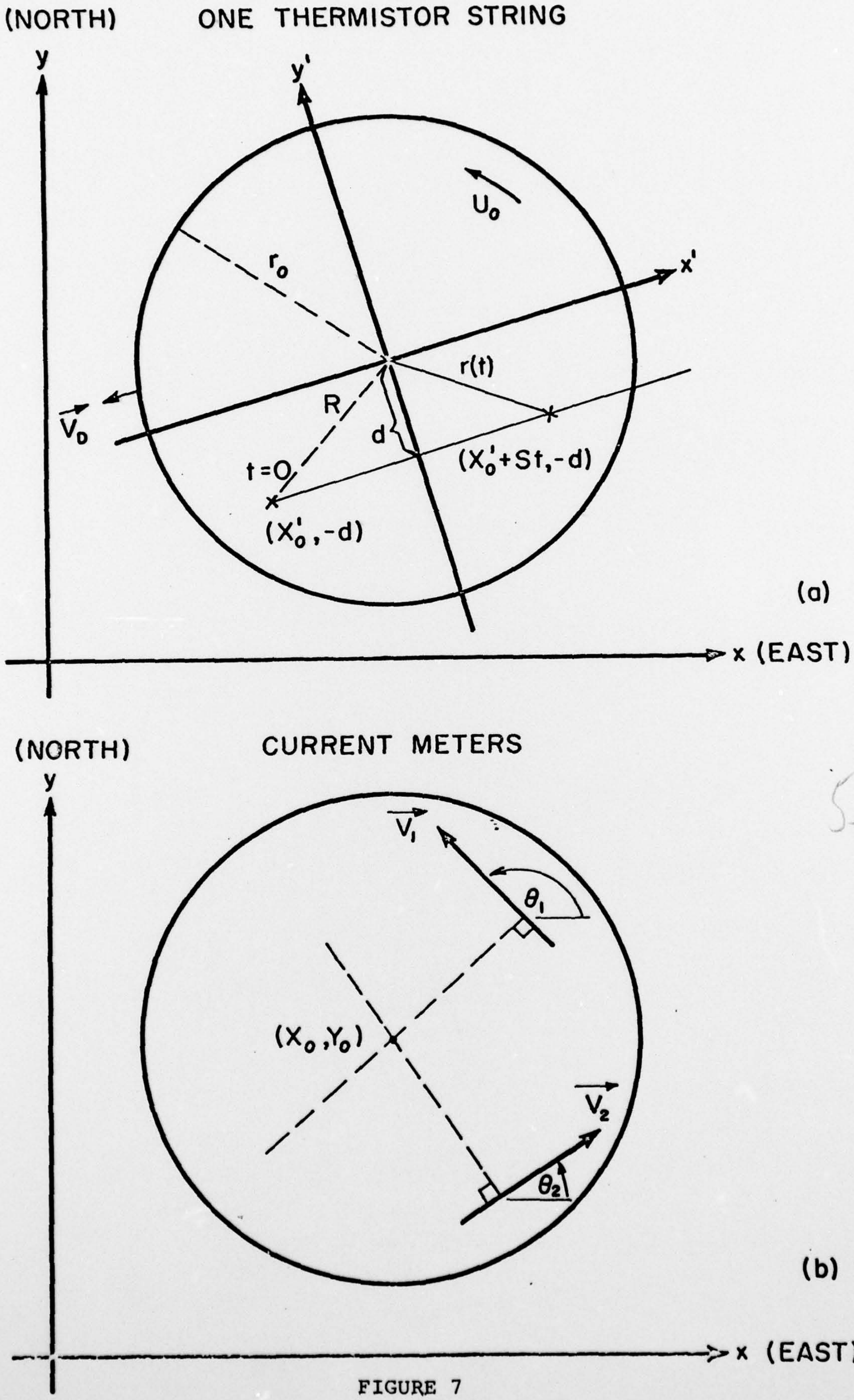
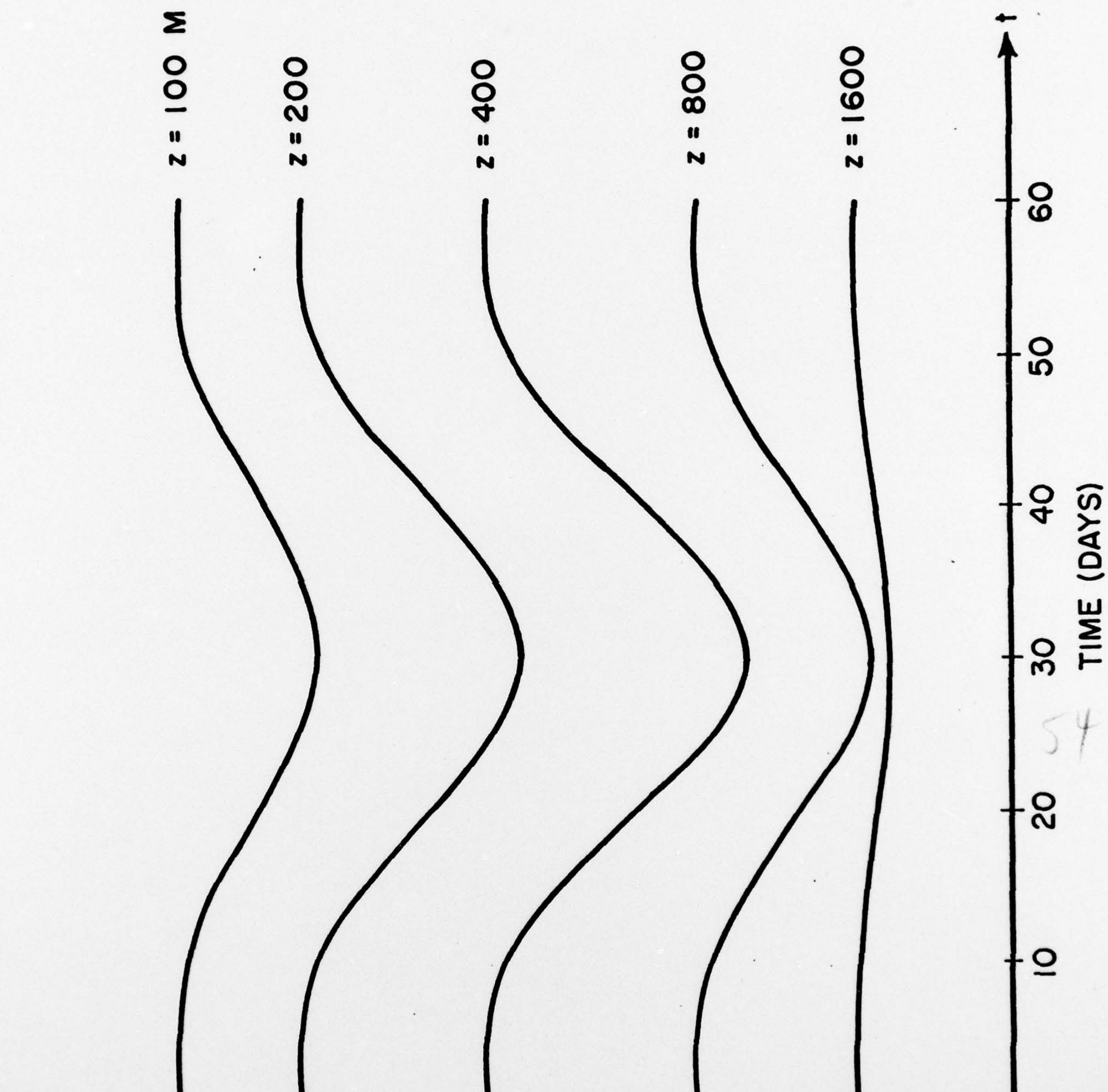
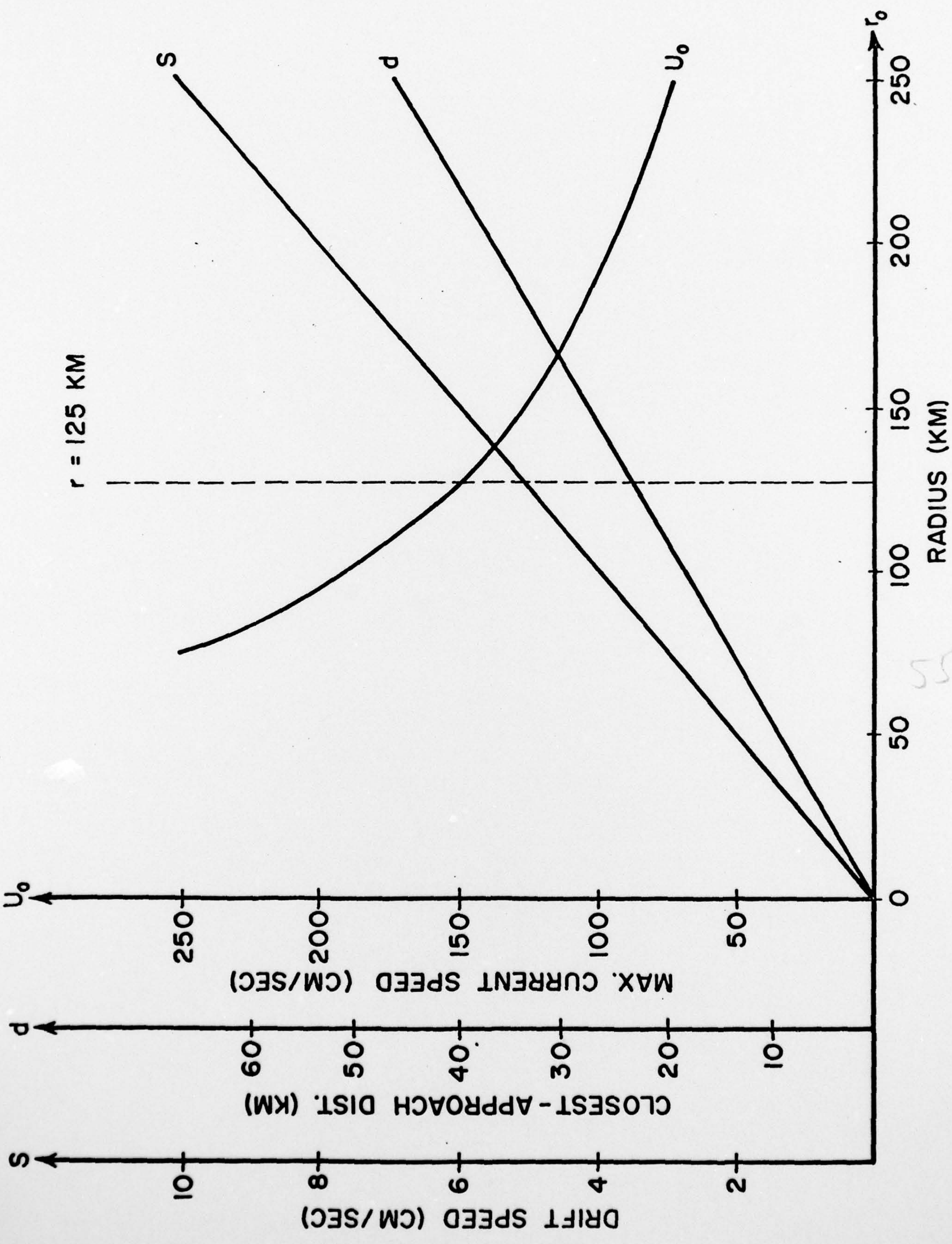

(NORTH)
ONE THERMISTOR STRING
y
y'
U_0
r_0
x'
r(t)
$\vec{V_0}$
R
d
t=0
$(X_0'+St,-d)$
$(X_0',-d)$
(a)
x (EAST)
(NORTH)
CURRENT METERS
y
$\vec{V_1}$
θ_1
(X_0,Y_0)
$\vec{V_2}$
θ_2
(b)
x (EAST)

FIGURE 7


z = 100 M
z = 200
z = 400
z = 800
z = 1600
T
t
TEMPERATURE (°C)
TIME (DAYS)
0
5
10
15
20
0
10
20
30
40
50
60

54

FIGURE 8



S
d
U_o
r = 125 KM
DRIFT SPEED (CM/SEC)
10
8
6
4
2
CLOSEST-APPROACH DIST. (KM)
60
50
40
30
20
10
MAX. CURRENT SPEED (CM/SEC)
250
200
150
100
50
0
50
100
150
200
250
RADIUS (KM)
r_o

FIGURE 9

55

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Bathythermographs
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Geophysical Fluid Dynamics

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The use of analytical modeling in the study of oceanic eddies is considered. Limited observational data, in combination with eddy models, can be used to obtain analytical approximations to environmental effects, including current and temperature perturbations, throughout the eddy. Techniques which efficiently use discrete measurements are presented to accurately specify any given analytical model containing an arbitrary number of (over please)

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parameters to an observed eddy. Questions of unique parameter specification and data sufficiency are considered for various data types and amounts, using a previously-derived eddy model. Examples with bathythermograph data are presented, in which eddy size, strength, and center position are to be determined. AXBT data is emphasized, and an investigation is made of the influence of the number of such instruments on the accuracy of parameter estimates. It is then shown how data obtained from oceanographic moorings can also lead to specification of eddy drift speed and direction. In both the bathythermograph and mooring examples, it is demonstrated that, even when the type of data available leads to nonunique parameter specification, significant information can be obtained about the observed eddy. Results in this paper suggest certain efficiencies in data utilization and in the design of subsequent experiments.